General Equ. of 2nd degree

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

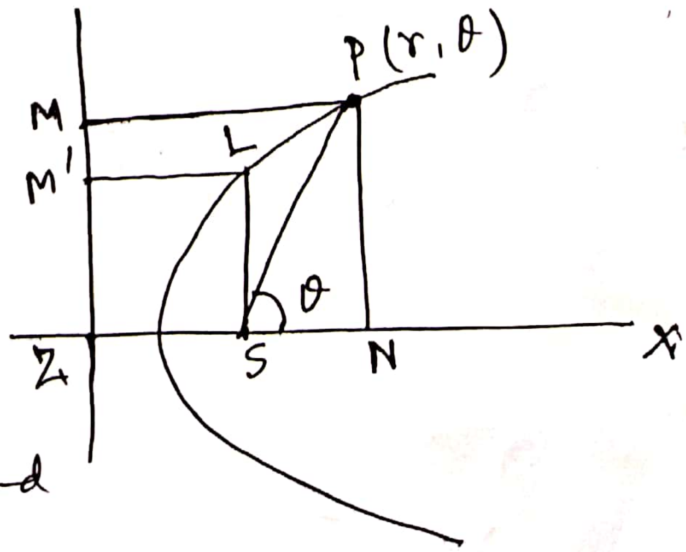
$$\Delta \equiv \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

1. when $\Delta = 0$; pair of st. lines
2. when $\Delta = 0$; $ab = h^2$; pair of parallel lines.
3. when $\Delta = 0$; $a+b=0$ " " perpendicular lines.
4. when $a=b$; $h=0$; a circle.
5. when $\Delta \neq 0$, $ab - h^2 = 0$; a parabola
6. when $\Delta \neq 0$; $ab - h^2 > 0$; an Ellipse.
7. when $\Delta \neq 0$; $ab - h^2 < 0$; ~~an~~ a hyperbola.
8. when $\Delta \neq 0$; $ab - h^2 < 0$ and $a+b=0$; a rectangular hyperbola.

Art 46 Polar equ. of a conic, focus being the pole.

$P(r, \theta)$

$SP = r$, $\angle XSP = \theta$.



Length of the semi-latus rectum be l, eccentricity be e.

PN & PM are drawn perp. to the Initial line and directrix.

$$e = \frac{SP}{PM}$$

$$\therefore SP = e\,PM = e\,ZN = e(ZS + SN)$$

Scanned with CamScanner

$$SP = e \cdot ZS + e \cdot SP\cos\theta$$

$$= e \cdot \frac{l}{e} + er\cos\theta$$

$$r = l + er\cos\theta$$

$$r(1 - e\cos\theta) = l$$

$$r = \frac{l}{1 - e\cos\theta} \quad \text{——— (1)}$$

ΔSPN,

$$\cos\theta = \frac{SN}{PS}$$

$$SN = PS \cdot \cos\theta$$

$$\frac{LS}{LM'} = e$$

$$LS = eLM'$$

$$= eZS$$

$$\therefore ZS = \frac{LS}{e} = \frac{l}{e}$$

If the initial line be SZ, then (1) becomes

$$r = \frac{l}{1 - e\cos(\pi - \theta)}$$

$$r = \frac{l}{1 + e\cos\theta}.$$

Relation between Direction cosines & Direction ratios

If a, b, c are the direction ratios, then $a \propto l$, $b \propto m$ & $c \propto n$

$\therefore a = kl,\ b = km,\ c = kn.$

we know that $l^2 + m^2 + n^2 = 1$ | $l = \frac{a}{k}$, $m = \frac{b}{k}$, $n = \frac{c}{k}$

$$\frac{a^2}{k^2} + \frac{b^2}{k^2} + \frac{c^2}{k^2} = 1$$

$$\therefore a^2 + b^2 + c^2 = k^2 \qquad \therefore k = \pm\sqrt{a^2+b^2+c^2}$$

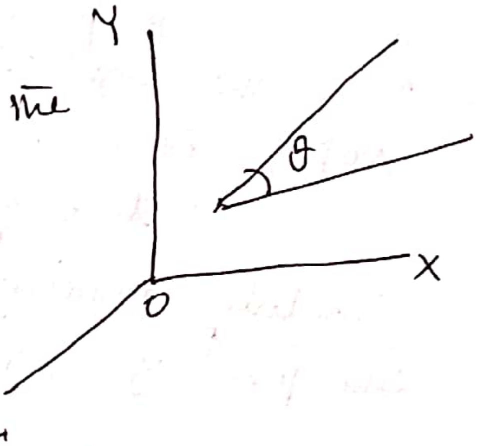
Now $l = \frac{a}{k}$

$$l = \frac{a}{\pm\sqrt{a^2+b^2+c^2}},$$

Similarly $m = \frac{b}{\pm\sqrt{a^2+b^2+c^2}}$ & $n = \frac{c}{\pm\sqrt{a^2+b^2+c^2}}$.

Angle between two lines: Let us consider two lines on 3 dimensional space whose direction cosines are l_1, m_1, n_1 & l_2, m_2, n_2 respectively. If the angle between the lines be θ, then

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2.$$



Q: Find the direction cosines of a lines which makes equal angles with the axes.

A:– Let l, m, n be the dcs of the line. then $l = \cos\alpha$, $m = \cos\beta$ & $n = \cos\gamma$.

In this case $\alpha = \beta = \gamma$, then we can write the dcs as $\cos\alpha, \cos\alpha, \cos\alpha$.

we know $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Here $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$3\cos^2\alpha = 1$

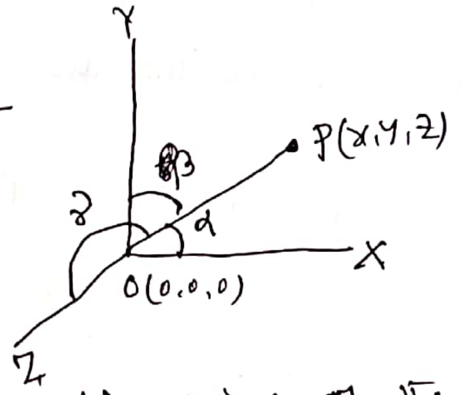
$\cos^2\alpha = \frac{1}{3}$

$\therefore \cos\alpha = \frac{1}{\pm\sqrt{3}}$

Hence direction cosines are $\frac{1}{\pm\sqrt{3}}, \frac{1}{\pm\sqrt{3}}, \frac{1}{\pm\sqrt{3}}$ Ans.

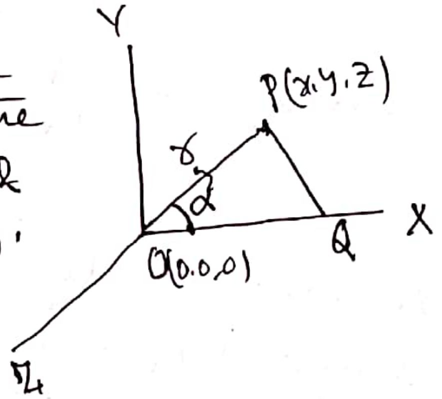
Rectangular Axes

Direction cosines: consider a point $P(x,y,z)$ on 3-dimensional space. OP is joined. The line OP makes angles α, β, γ with the positive directions of X, Y, Z axes. Then



$\cos\alpha$, $\cos\beta$, $\cos\gamma$ are known as Direction cosines of the line OP. Direction cosines are also denoted by l, m, n respectively.

A useful relation: If $P(x,y,z)$ is a point on space & l, m, n are the dcs of OP. Then $x = lr$, $y = mr$ & $z = nr$; where $r = OP$ (radius vector).



Draw PQ perpendicular to x-axis. Then the right angled triangle $\triangle OPQ$,

$$\cos\alpha = \frac{OQ}{OP}$$

$$l = \frac{x}{r} \qquad \therefore x = lr$$

Similarly drawing perpendiculars to Y & Z axes, we can prove $y = mr$ & $z = nr$.

Now

$$OP^2 = (0-x)^2 + (0-y)^2 + (0-z)^2$$

$$r^2 = x^2+y^2+z^2$$

$$r^2 = l^2r^2 + m^2r^2 + n^2r^2.$$

$$r^2 = r^2(l^2+m^2+n^2).$$

$$\therefore l^2+m^2+n^2 = 1.$$

That is $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$

Direction ratios: The quantities which are proportional to direction cosines are known as Direction ratios. If $a \propto l$, $b \propto m$ & $c \propto n$, then a, b, c are the direction ratios.